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An experiment on the flow past a finite circular cylinder at high subcritical and supercritical Reynolds numbers

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The complicated flow in the tip region of a finite circular cylinder in uniform cross flow has been examined at the Reynolds numbers $0{\cdot}85\times10^5$, $1{\cdot}8\times10^5$, and $7{\cdot}7\times10^5$. Simultaneous measurements of the surface-pressure and wake-velocity fluctuations have revealed the existence of a shedding regime in the tip region that is distinct from the one prevailing on the main body of the cylinder. In particular, this regime can be unstable and intermittent, can have a cellular structure in the wake, or can be subcritical when the main flow is supercritical.

1. Introduction

The flow past an infinite cylinder at low speeds with uniform upstream conditions has been the subject of extensive and widely varied type of research for many years. As a result, a considerable amount of knowledge, concerning this particular flow, has been accumulated which, although incomplete and far from satisfactory, allows a certain degree of confidence in dealing with related engineering problems. One of the cases that often occur in applications and for which this body of knowledge has provided valuable guidance is that of a finite cylinder with one free end in a cross flow. Here, if the aspect ratio is large enough, the structure of the flow away from the tip of the cylinder is qualitatively the same as it would be if the cylinder were infinitely long. Thus the presence of a free end totally immersed in the flow was generally considered in the past as a minor factor in the overall flow picture and one that would require special attention only in specific cases of engineering concern. This resulted, inevitably, in a general neglect of the problem as one of fundamental interest while at the same time causing, in view of the practical importance of such a problem, a great number of investigations of an applied nature to be carried out on a wide variety of configurations of immediate engineering interest. As a result, what we have today is a very limited number of published papers on the fundamental nature of the flow and a considerable amount of information and data, which are difficult to correlate and which are scattered in numerous agency reports and the proceedings of several conferences and specialized meetings.

It is not our intention here to present a comprehensive review of all the disparate works that have been performed on the subject; these can be found in the proceedings of the various conferences on 'Wind Effects on Buildings and Structures' and 'Flow-Induced Structural Vibrations' and, the most up-to-date, in the proceedings of two such conferences (see Naudascher 1974; Eaton 1977). Our intention, rather, is to review briefly those experimental results found in the literature that hold special clues to the fundamental nature of the flow. The present work itself is an attempt at obtaining further information of this type.

Perhaps the first question that comes to mind in the present context is how the familiar vortex street found behind infinite cylinders is altered when the cylinder is no longer infinite but has one end totally immersed in the flow. The vortex filaments, not being able to extend beyond the cylinder's tip, should obviously terminate somehow in the fluid. The first and the only work that addressed this issue was by Taneda (1952). He concluded, on the basis of a flow-visualization experiment done at low Reynolds numbers (below 100), that each vortex in one row connects to the two facing vortices in the opposite row. While one expects the situation to be much more complicated at higher Reynolds numbers, and there is ample evidence to that effect, it is hard to accept the suggestion made sometimes in the literature and which is based on smoke pictures (Maull & Young 1974) or inferred from surface oil patterns or other partial modes of visualization (Gould, Rymer & Ponsford 1968; Etzold & Fiedler 1976) that there exists a symmetrical pair of vortices that spring from the tip, in the downstream direction. The existence of a fluctuating lift on the cylinder, which, incidentally, was found by Etzold & Fiedler (1976) to have a maximum near the tip, rules out any such symmetrical arrangement there.

The frequency of vortex shedding, when such shedding occurs, is another issue that received some attention in the literature. The most comprehensive set of data on the variation of this frequency with Reynolds number and aspect ratio, and for a given Reynolds number and a given aspect ratio, its variation in the spanwise direction, was obtained by Okamoto & Yagita (1973). These data show that the shedding frequency (non-dimensionalized) decreases gradually as one approaches the tip, while on the main portion of the cylinder where it is practically uniform it decreases with both decreasing Reynolds number and aspect ratio. However, these data are limited to a relatively small range of low values of Reynolds numbers ($1{\cdot}7 \times 10^3$–$1{\cdot}5 \times 10^4$) and to a range of aspect ratios between 7 and 20. Available data at high Reynolds numbers, in the range of most cases of practical interest, are much less systematic and often exhibit peculiar behaviour. Fiedler & Wille (1970) observed a distinct frequency component, with a Strouhal number of 0·45, in the velocity fluctuations near the tip of a short cylinder of aspect ratio 4, at a critical Reynolds number. This frequency component became weaker as the Reynolds number was decreased. Wootton (1968) observed regular lateral vibrations of his model at supercritical Reynolds numbers, at a Strouhal number characteristic of subcritical values. He attributed this to the fact that his model is free to vibrate. But, as we will see later in this work, such behaviour is not peculiar only to vibrating cylinders but is inherent to the nature of the flow past a finite cylinder in general.

In order to explain the above observations on the variation of the shedding frequency with aspect ratio and Reynolds number, and its variation in the spanwise direction, one is tempted to use the concept of a universal Strouhal number that relates the shedding frequency to the base pressure and the wake width (Bearman 1967). Indeed Okamoto & Yagita measured, at a fixed Reynolds number, the base pressure on the main portion of the cylinder for several values of the aspect ratio and the results show an increase in the base pressure attending a decrease in the aspect ratio, in harmony with the decrease in the shedding frequency. However, this conformity with the idea of a universal Strouhal number breaks down near the tip, as is revealed by the data on the spanwise behaviour of these quantities. At the National Physical Laboratory (see Wootton 1968) the gradual decrease in the shedding frequency near the tip was thought to be due to a widening of the wake there. On the other hand Okamoto & Yagita found a slightly narrower wake near the tip.

It is clear that the concept of a universal Strouhal number, which was established on the basis of data taken with infinite cylinders, cannot be used in regions of highly three-dimensional flow such as near the tip of a finite cylinder. Neither does it seem possible to explain at this stage the anomalous behaviour, mentioned above, at critical and supercritical Reynolds numbers. What is also intriguing is the observed gradual decrease in the shedding frequency towards the tip. It is well-known that when the frequency of vortex shedding changes in the spanwise direction, as for instance when shear is present in the free stream (see Maull & Young 1973) the flow divides itself into separate cells and the change occurs in steps rather than in a continuous manner.

It is clear from the above discussion that there are important unresolved questions about the flow past a finite cylinder, particularly in the vicinity of the free end and at high values of the Reynolds number. In the present work an attempt is made at answering some of these questions by analysing the results of an experiment during which data on the surface-pressure fluctuations near the tip of a finite circular cylinder and on the velocity fluctuations in the wake were taken simultaneously at high subcritical and supercritical values of the Reynolds number. Originally, the work was motivated by the problem of sound radiation from the landing gear of an aircraft. During landing and take-off the landing gear can be thought of as a finite strut protruding from a large surface, and typical values of the Reynolds number based on the strut diameter (for a Boeing 747) are of the order of 10^6, a supercritical value. There are also indications, based on work done at the Jet Propulsion Laboratory (see Kendall 1977), that the region near the tip of the strut is one of relatively intense noise generation. Thus the objective of the present work was to provide some useful information about the aerodynamic noise problem.

2. Experimental set-up and procedure

The test was conducted in the 7 ft × 10 ft low-speed closed-circuit wind tunnel at the N.A.S.A. Ames Research Center. The model, a right finite circular cylinder 71·75 in. long and 6 in. in diameter, was mounted normally on the wall of the test section, parallel to the long dimension of its cross-section, and was sealed at the free end. Ten B&K $\frac{1}{2}$ in. condenser microphones, type 4133, were mounted flush with the surface of the cylinder near the tip and were used as surface-pressure transducers. One of these microphones was mounted on the flat end, the other nine were distributed along two generators 180° apart; the distance between two consecutive microphones in each row was 4·5 in. or three quarters of a diameter. The model could be rotated in steps of 45°, thus allowing measurements of the surface pressure at different angular positions to the wind to be made. Figure 1 shows the model in the test section as well as the location of the microphones, numbered, near the tip. Also shown are the three angular positions to the wind that the model occupied in the course of the test. For wake-velocity measurements, two standard DISA single hot-wire anemometers were used. One hot-wire probe was fixed near the tip of the cylinder and was used as a reference probe, the other occupied successively 19 different positions in the wake. One of these positions

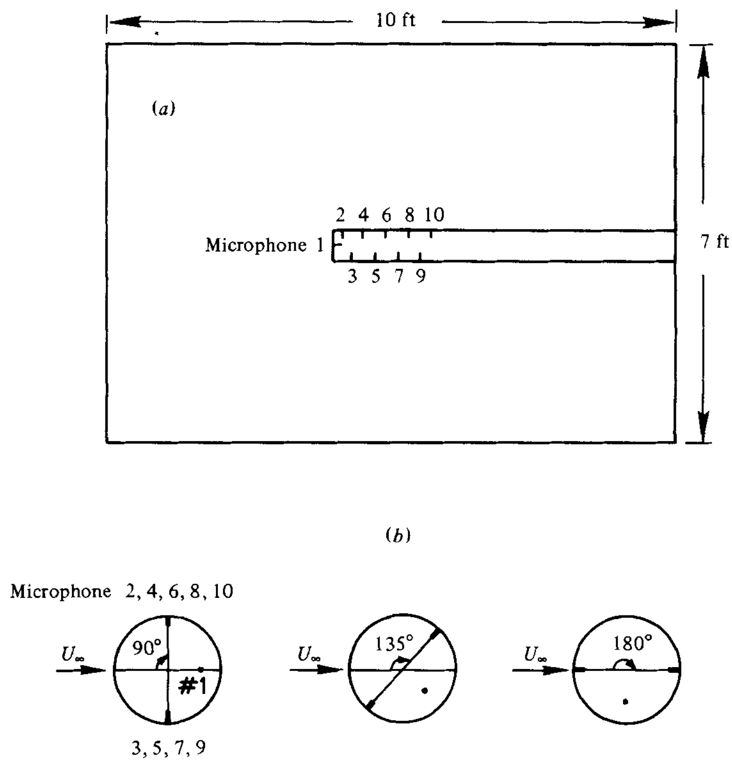


FIGURE 1. (*a*) The model in the test section and the location of the microphones; (2) at 1·75 in. from the tip; (3) at 3·25 in. from the tip; distance between two consecutive microphones is 4·5 in. (*b*) The three angular positions to the wind tested.

was a mirror image of the reference probe's position with respect to the wake centre plane, the other 18 were five diameters downstream of the cylinder. Figure 2 depicts this arrangement.

The ten microphones were calibrated using a B&K pistonphone, a set of variable-gain amplifiers, and an r.m.s. meter. It should be noted, however, that the frequency response of the $\frac{1}{2}$ in. condenser microphones is not uniform down to low frequencies, in the range of interest for the present experiment, and the roll-off of the frequency response curve is not exactly the same for all the microphones used. The non-uniformity, which is of the order of 1 db at 10 Hz, was not, however, judged significant enough to warrant a painstaking correction, and in any case does not affect the interpretation of the results, as we will see later. Only fluctuating quantities were measured during the experiment. This applies to both the surface pressure and the velocity in the wake. Simultaneous measurements of the surface pressure and the wake velocity were made only when the microphones were at 90° to the wind.

Wind-tunnel runs were made at three different values of the free-stream dynamic pressure q; q is defined as $\frac{1}{2}\rho U_\infty$, where ρ and U_∞ are respectively the fluid density and the velocity in the free stream. These values are 1 lbf/ft^2, 4 lbf/ft^2, and 80 lbf/ft^2 and correspond roughly to three different values of the Reynolds number R; R is defined as $\rho U_\infty d/\mu$, where μ and d are respectively the fluid viscosity and the cylinder diameter. Since the test had to be conducted continuously over a period of several days, there

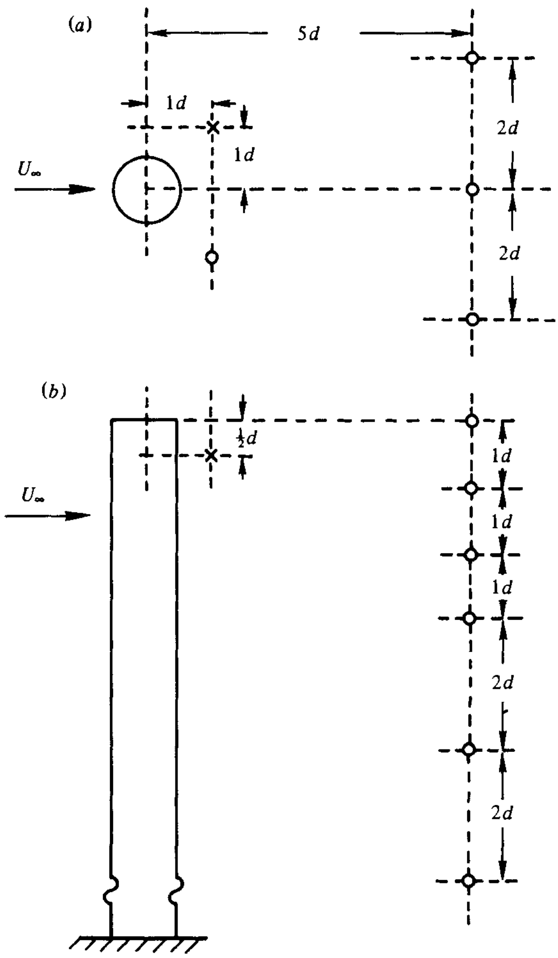


FIGURE 2. Hot-wire-probe positions in the wake. ×, reference probe; ○, traversing probe; d, cylinder diameter. (a) Axial view; (b) top view. In all positions the probe wires were parallel to the cylinder.

were significant changes in the ambient temperature between different wind-tunnel runs, particularly from early morning to afternoon, which resulted in different values of R at a given value of q. However, R was evaluated at each run, using a standard chart of R versus q, temperature, and static pressure in the free stream, and the changes were taken into account in the data analysis and the interpretation of the results. On the average, the three values of R thus studied are $0{\cdot}85 \times 10^5$, $1{\cdot}80 \times 10^5$, and $7{\cdot}7 \times 10^5$. During each run, signals from the ten microphones or, when simultaneous measurements of pressure and velocity were made, from the ten microphones and the two hot-wire probes were recorded on a 14-channel magnetic tape, using an Ampex model 1300A tape recorder, for later analysis.

It should be noted that, because of the nature of the facility and instruments used, and other considerations as well, measurements at lower values of R than those mentioned above could not be made, however desirable that may have been. In fact, the measurements at the lowest value, viz $R = 0{\cdot}85 \times 10^5$, were thought at the time of the test to be unreliable in view of the very low value of q required. The appearance of the surface-pressure signals, which were constantly monitored during the experiment, as well as previous experience with the N.A.S.A. Ames 7 ft × 10 ft wind tunnel (the tunnel is vented to the outside at the fan section, where wind and other atmospheric conditions could affect the flow inside) led us to conclude that the free-stream dynamic pressure, at this particular Reynolds number, was too low to have a steady uniform flow in the test section, and consequently a decision was made not to record a complete set of data at $R = 0{\cdot}85 \times 10^5$. However, it was found out later, when it was no longer possible to complete the data, that what was thought to be due to unsteadiness in the free stream was actually due to a low-frequency component in the surface-pressure fluctuations. Thus the available results at $R = 0{\cdot}85 \times 10^5$ are limited and do not include information about the velocity field. At $R = 1{\cdot}80 \times 10^5$ the r.m.s. level of the turbulence in the free stream was about 0·5 %, while at $R = 7{\cdot}7 \times 10^5$ it was about 1·1 %.

The analysis of the data consisted simply of first generating autocorrelation functions and frequency spectra, using a Honeywell model SAI 43A correlator in conjunction with a Honeywell model SAI-470 Fourier transform analyzer, of all pressure and velocity signals. These autocorrelation functions and frequency spectra were then examined carefully, together with the unprocessed signals themselves; and various filtering and cross-correlation operations followed as was judged appropriate.

It is perhaps appropriate at this stage, and before introducing the results of the test, to emphasize that no attempt was made in the present work to study (except when noted) the effect of small departures in the experimental conditions from the ideal or nominal ones, or the effect of small changes in the nature of these perturbations on the phenomena that we discuss in § 3. Such sensitivity studies have been a common practice in the past when flows past bluff cylinders and, in particular, circular ones were investigated; the flow past a circular cylinder in the critical and supercritical ranges has been found to be very sensitive to experimental conditions (see e.g. Bearman 1969). However, a special effort was made to ensure that the perturbations present were not of such a level as to introduce qualitative changes in the character of the corresponding basic flow past a cylinder spanning the whole wind tunnel. For instance, vibrations in the model were reduced to a small level, a very small fraction of the cylinder diameter, by means of additional reinforcements on the other side of the tunnel wall, and checked by means of a short-range telescope through which they were visible only at the highest value of R tested. Also the level of turbulence in the free stream and that of surface roughness of the model (64 microinches above mean surface) were such that the flow at $R \simeq 1{\cdot}80 \times 10^5$ was a genuine subcritical one, as will also become clear in § 3·2. Finally, the model's aspect ratio (nearly 12) and the boundary-layer thickness (of the order of two inches) along the wall containing the built-in end of the model were such that the phenomena inherent to the flow in the tip region could be considered at this stage of our knowledge free of any direct influence from the conditions at the wall. We also note that no special study of the acoustical environment of the tunnel was made. However, we did make sure that the sound components that might be present in the test section due to the rotating motion of the fan blades had frequencies different from the frequencies of the aerodynamic components that are the subject of our analysis in § 3; the blade-passage frequencies were $3{\cdot}70 \pm 0{\cdot}13$ Hz, $7{\cdot}6 \pm 0{\cdot}67$ Hz, and $32{\cdot}53 \pm 2$ Hz at $R = 0{\cdot}85 \times 10^5$, $1{\cdot}80 \times 10^5$, and $7{\cdot}7 \times 10^5$ respectively.

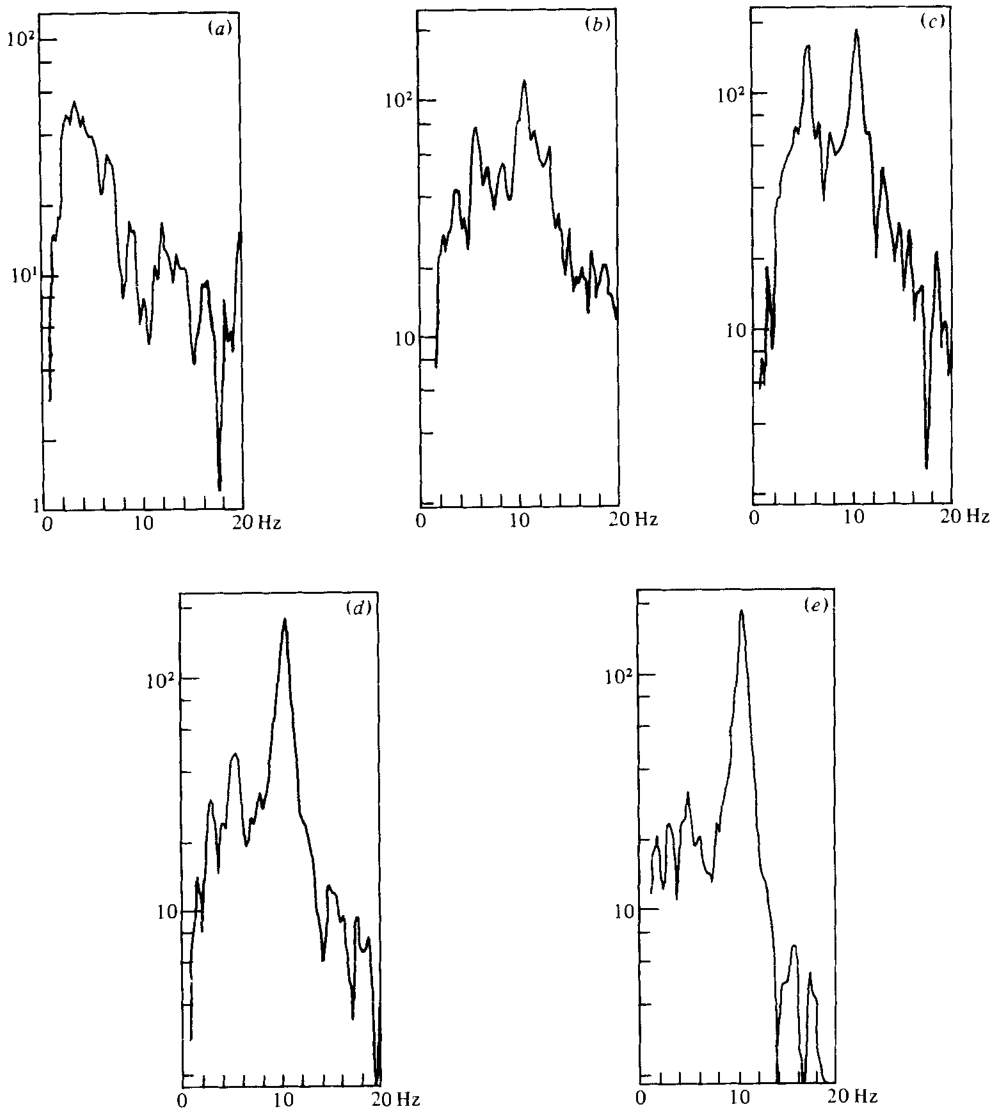


FIGURE 3. Frequency spectra of surface-pressure fluctuations at 135° to the wind and (*a*) 0·29*d*, (*b*) 1·04*d*, (*c*) 1·79*d*, (*d*) 2·54*d*, (*e*) 3·29*d* from the tip. The unit on the vertical axis is arbitrary but is the same for all spectra in figures 3, 4 and 5. $R = 0{\cdot}85 \times 10^5$.

3. Results and discussion

In the following, the results of the data analysis will be introduced and discussed for each value of R separately, and an effort will be made to restrict the presentation to only those results that are pertinent to the discussion. General conclusions and recommendations will be made in § 4.

3.1. *The case* $R = 0{\cdot}85 \times 10^5$ ($q = 1\,\mathrm{lbf/ft^2}$)

Data from only one wind tunnel run are available at $q = 1\,\mathrm{lbf/ft^2}$, and therefore only one value of R applies in this case. This value, $R = 0{\cdot}85 \times 10^5$, is a subcritical one by

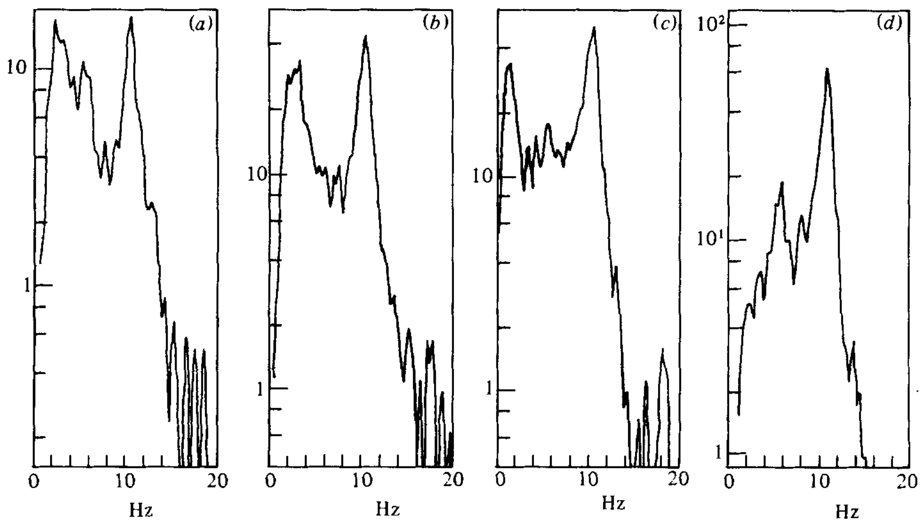


FIGURE 4. Frequency spectra of surface-pressure fluctuations at 45° to the wind and (a) 0·60d, (b) 1·35d, (c) 2·10d, (d) 2·85d from the tip. $R = 0{\cdot}85 \times 10^5$.

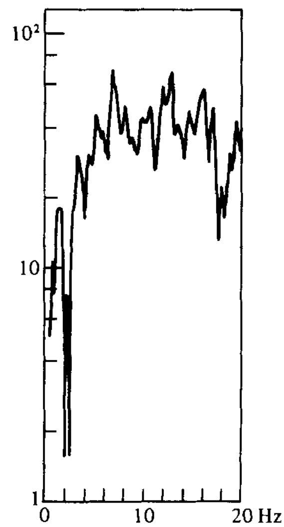


FIGURE 5. Frequency spectrum of surface-pressure fluctuations on the flat end of the cylinder. $R = 0{\cdot}85 \times 10^5$.

infinite-cylinder standards and the expected vortex-shedding frequency, i.e. the shedding frequency that would be observed if the cylinder were infinite, is about 11·3 Hz.† Figures 3 and 4 show respectively the frequency spectra of the surface-pressure fluctuations at 135° and 45° to the wind, while figure 5 shows the frequency spectrum of these fluctuations on the flat end of the cylinder, at the position of microphone 1 that corresponds to the second angular position of the model (see figure 1*b*). The spectra

† This figure was arrived at by using a value of the Strouhal number S of 0·185 (see Roshko 1961, figure 5) and the identity $f = 2qS(32{\cdot}16/\mu R)$, where f is the shedding frequency and μ is a function only of temperature. It is a rough value since available data on S for infinite cylinders show some scatter, which is due to the sensitivity of the flow to experimental conditions.

shown are portions of power spectra that were computed in the frequency range between 0 and 100 Hz, with a frequency resolution of 0·5 Hz. An interpolation scheme built in the Fourier transform analyzer allowed, in addition, an accurate reading of 0·1 Hz of the frequency corresponding to any spectral peak. However, the values which will be quoted below are all rounded off to the nearest 0·5 Hz.

A distinct spectral peak at 10·5 Hz is evident in the spectra of figures 3 and 4 (actually all the peaks in question are at 10·6 Hz except the one in figure 4 (*b*), which is at 10·5 Hz). Its magnitude increases with increasing distance from the tip and is smaller at 45° than at 135°. A check on the nature of the surface-pressure component that corresponds to this peak was made by cross-correlating the signal from microphone 10, after band-pass filtering between 8 and 12 Hz, successively with the filtered signals from microphones 9 to 2, and it showed that this component is due to a vortex-shedding process; a 180° phase shift between the pressure signals on the upper side of the cylinder and those on the lower side with respect to the free stream was clear in the cross-correlation functions, except in the case of the signal from microphone 2 where no correlation was evident.† That the magnitude of the peaks is smaller at 45° than at 135° is consistent with the state of affairs that exists in the case of an infinite cylinder, namely the fact that the amplitude of the surface-pressure fluctuations at the shedding frequency is smaller on the front side of the cylinder than on the rear side (see Gerrard 1961; Ayoub & Karamcheti 1976). Now, whether the slight difference between the 10·5 Hz observed here and the 11·3 Hz mentioned above is due to the effect of the finiteness of the cylinder or not is hard to say; no measurements were made, in the present work, with the cylinder spanning the whole wind tunnel. However, the important thing to note is that the peaks in the spectra of figures 3 and 4 that correspond to vortex shedding are all at 10·5 Hz. This means that the vortices are initially shed at one single frequency up to the tip of the cylinder, and suggests that, possibly, the gradual decrease in the frequency toward the tip observed by, for instance, Okamoto & Yagita (1973) and, for that matter, in the present work at $R = 1{\cdot}80 \times 10^5$, where wake-velocity measurements were made, is due to a separate phenomenon occurring downstream of the formation region. A spanwise variation, at some distance downstream, in the frequency of passage of vortices can only occur if streamwise vortices form and the flow divides itself into separate cells, within each of which the frequency is constant. Could this be what is happening here?

One of the referees has indicated that perhaps the observed gradual decrease in the frequency toward the tip, as opposed to a stepwise change, is due to an up-and-down motion of the cells along the cylinder, resulting in an 'average' frequency being detected by the hot wire at a fixed spanwise position. Indeed, this is a plausible explanation, since, as we will see in § 3.2, there is evidence of a fluctuating spanwise motion in the wake of the cylinder tip.

† One referee has correctly pointed out that a phase difference of 180° across the 45°–135° diameter does not necessarily prove that the component in question is due to an asymmetric process of vortex shedding: such a phase relationship could equally be due to a symmetric-shedding process. However, we feel that the likelihood of such a symmetric shedding in the present experiment and at the Reynolds number under consideration is very remote, given the fact that at the other Reynolds numbers asymmetric shedding occurred. In addition, we see no reason why such shedding would occur under the present conditions: symmetric vortex shedding is known to occur only when the cylinder is oscillated in the free-stream direction at appropriate frequencies.

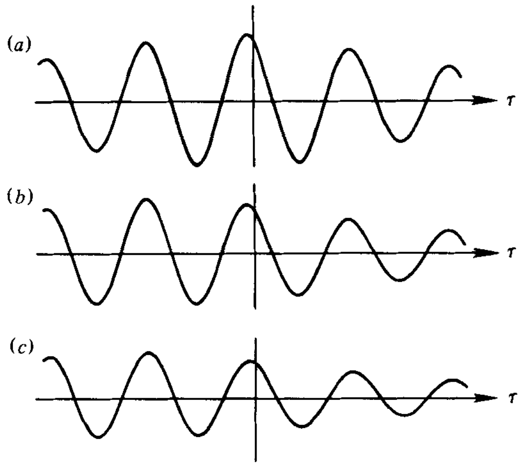


FIGURE 6. Cross-correlation functions of filtered signals from (a) microphones 10 and 8, (b) microphones 10 and 6, (c) microphones 10 and 4; τ is the time lag. Signal from microphone 10 was delayed. Microphones are at 135° to the wind. $R = 0{\cdot}85 \times 10^5$.

Another distinct peak at 5·5 Hz is evident in the spectra of figures 3 and 4. Its magnitude increases with distance from the tip, seems to reach a maximum at $1{\cdot}79d$, where it is comparable to the magnitude of the 10·5 Hz component, and then tapers off farther along the span. Filtering and cross-correlation operations showed that this peak does not correspond to a vortex-shedding process. In fact, it was found that a phase difference of 90° exists between the pressure signals from the upper and lower sides of the cylinder, with the ones from the upper side leading. No explanation for this can be given at this stage.

In addition to the two frequency components discussed above the pressure fluctuations on the surface of the cylinder seem to contain, as figures 3 and 4 show clearly, low- and high-frequency irregular fluctuations which are most energetic near the tip. As for the pressure fluctuations on the flat end, figure 5 shows that they are highly irregular and of relatively high intensity.

One last feature is worth mentioning here. It concerns the nature of the cross-correlation functions that were computed for the purpose of determining the phase difference between the pressure fluctuations, at the shedding frequency, at different positions on the cylinder. As figure 6 shows, the cross-correlation function of the signal from microphone 10 with that from microphone 8 is maximum at about zero time separation, the cross-correlation function of the signal from 10 with that from 6 is maximum at about minus one period, and finally the cross-correlation function of the signal from 10 with that from 4 is maximum at minus one or two periods. In addition, all three functions reveal higher correlation at negative time separations than at positive ones. Since in these cross-correlations the signal from microphone 10 was delayed, this means that the pressure pattern around the shedding frequency is moving in the spanwise direction away from the tip.

We would like to conclude this subsection by raising the possibility that the propagating motion indicated above might be related to the formation and motion of the vortex cells previously mentioned. If these cells do indeed form they must be carried by the downwash (flow toward the root) that is known to exist behind the

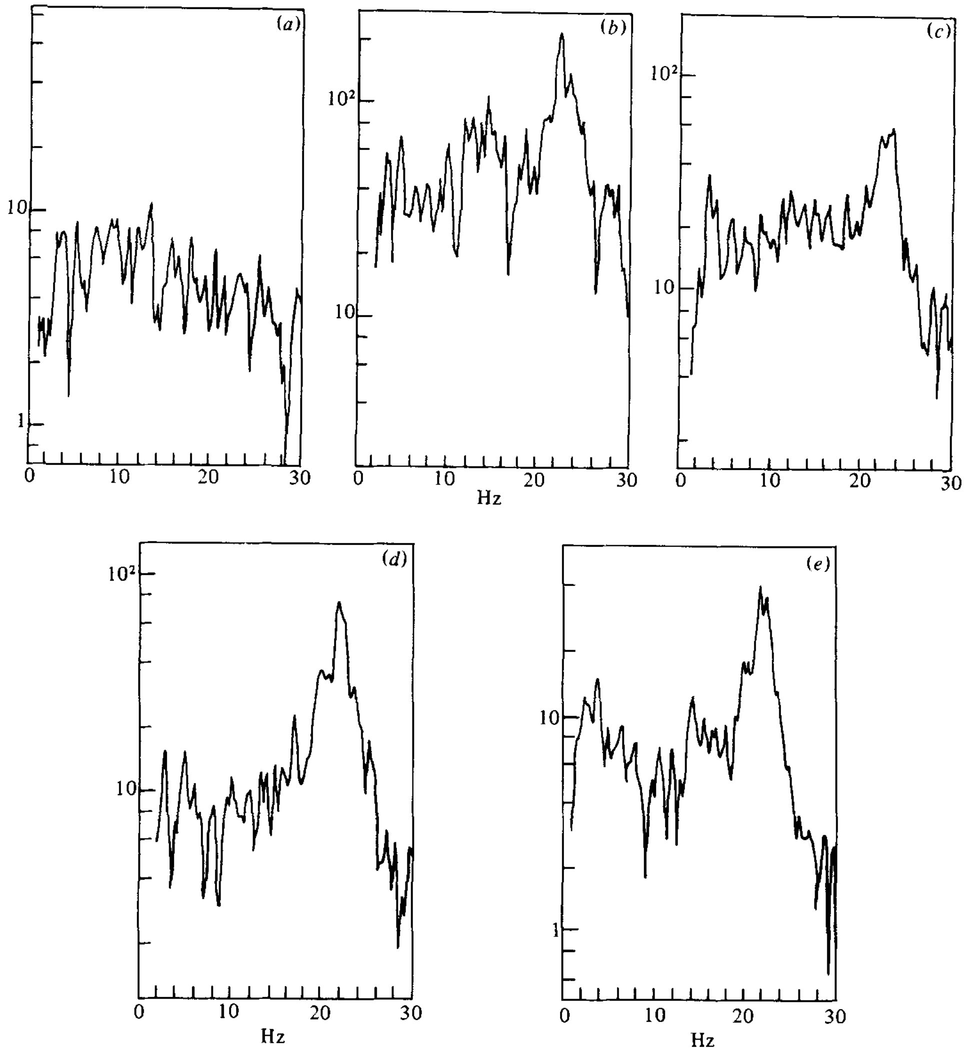


FIGURE 7. Frequency spectra of surface-pressure fluctuations at 135° to the wind and (*a*) 0·29*d*, (*b*) 1·04*d*, (*c*) 1·79*d*, (*d*) 2·54*d*, (*e*) 3·29*d* from the tip. The unit on the vertical axis is arbitrary but is the same for all spectra in figures 7, 8 and 9. $R \simeq 1{\cdot}8\times10^5$.

cylinder tip (see Etzold & Fiedler 1976). Naturally, no firm conclusions can be made before a more detailed examination of the flow in thc vicinity of the tip is undertaken.

3.2. *The case* $R \simeq 1{\cdot}8\times10^5$ ($q = 4\,\mathrm{lbf/ft^2}$)

The data recorded at $q = 4\,\mathrm{lbf/ft^2}$ correspond, for the reasons indicated in §2, to a range of R between about $1{\cdot}73\times10^5$ and $1{\cdot}86\times10^5$. The values in this small range are all subcritical by infinite-cylinder standards and narrow-band vortex shedding would be expected if the cylinder were infinite. The corresponding vortex-shedding frequencies would range from about 23·3 Hz to about 23·6 Hz. These values for the frequency were obtained using a Strouhal number of 0·2 – a value measured by Relf and Simmons in 1924 (see Bearman 1969) that we consider more appropriate for the present experiment, because of our level of turbulence in the free stream, than the values measured in a

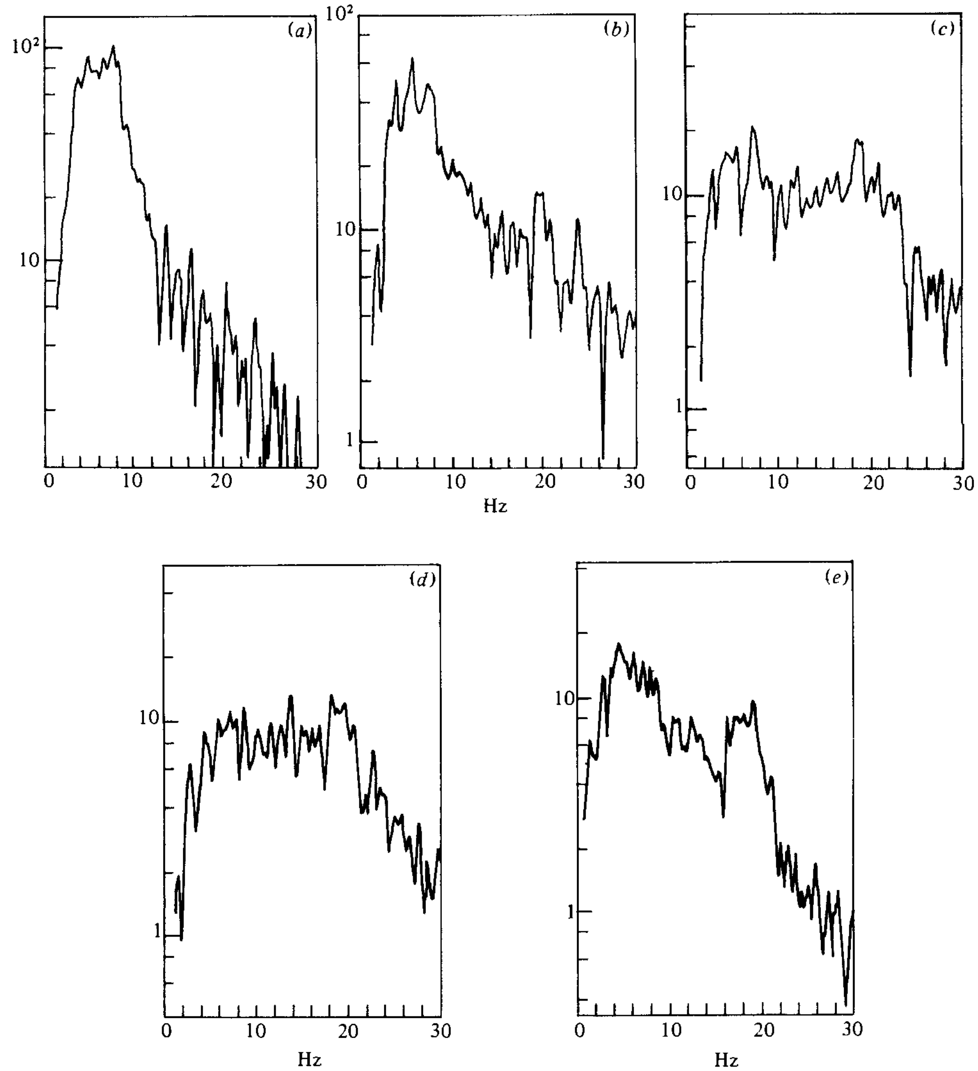


FIGURE 8. Frequency spectra of surface-pressure fluctuations at 90° to the wind and (*a*) 0·29*d*, (*b*) 1·04*d*, (*c*) 1·79*d*, (*d*) 2·54*d*, (*e*) 3·29*d* from the tip. $R \simeq 1{\cdot}8 \times 10^5$.

cleaner wind tunnel and with a smoother cylinder by, for instance, Bearman (1969). The appropriateness of this choice will also become evident later in this subsection when we introduce the results of the hot-wire measurements. The power spectra that will be discussed below were all computed in the range between 0 and 100 Hz with a frequency resolution of 0·5 Hz. Therefore the expected slight variations in the shedding frequency, resulting from variations in the conditions in the wind tunnel, will not be resolved in the spectra, and consequently will not affect our discussion.

Figures 7, 8, and 9 show respectively the frequency spectra of the surface-pressure fluctuations at 135°, 90°, and 45° to the wind. The first thing that is apparent in these spectra is that, unlike the situation at the previous Reynolds number, the frequency content of the surface-pressure fluctuations at the present Reynolds number is much more complex. Prominent isolated spectral peaks that could be associated with a single

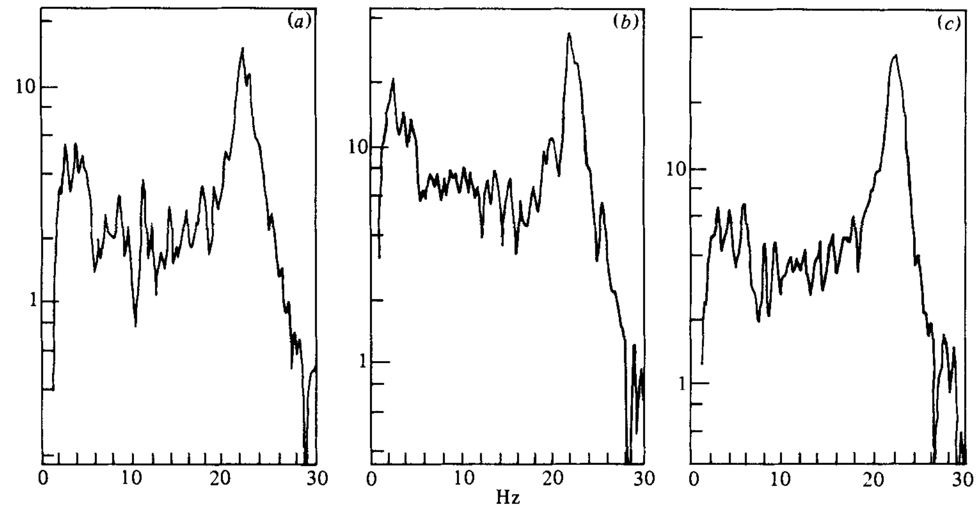


FIGURE 9. Frequency spectra of surface-pressure fluctuations at 45° to the wind and (*a*) 0·60*d*, 1·35*d*, (*c*) 2·85*d* from the tip. $R \simeq 1{\cdot}8 \times 10^5$.

frequency component do not appear. Instead, what we see, in most of these spectra, is a group of prominent peaks, or a prominent peak accompanied by side peaks. In order to investigate this question further, the raw signals, i.e. the unprocessed pressure signals, were examined simultaneously, two at a time, on a dual-beam oscilloscope. What was found was that the pressure signals do contain a periodic component. However, this component occurs only intermittently, is strongly modulated in amplitude, and occasionally strong spikes, positive followed by negative ones, occur and become more regular toward the tip. This last feature was more evident in the pressure signals at 90° to the wind and could account for the shape of the spectra in figure 8, since spikes are known to produce wide-band spectra.

In addition to the above, the visual inspection of the raw signals revealed that when the periodic component in the pressure fluctuations occurs it does so simultaneously at different locations on the cylinder and is about 180° out of phase between points on the upper and lower sides of the cylinder with respect to the free stream. This was also confirmed by filtering and cross-correlating the signals taken (simultaneously) at 45° and 135° to the wind; this time the signals were band-pass filtered between 20 and 25 Hz. Evidence, from the cross-correlation functions, of a propagating pressure pattern in the spanwise direction was much less in this case than in the previous one, that is at the lower value of *R*.

Figure 10 shows some samples of the pressure signals band-pass filtered between 20 and 25 Hz. The signals are shown in pairs; in each pair the upper and lower signals correspond respectively (except in figure 10*e*) to positions on the upper and lower sides of the cylinder with respect to the free stream. Figures 10(*a*, *b*) show sample pressures at 90° to the wind, while figures 10(*c*–*e*) show sample pressures at 135° and 45°. The signals were filtered in order to remove the background irregular components, so the characteristics of the 'periodic' component could be clearly seen.

One more important feature that is evident in the spectra of figures 7 and 9 is that the prominent peaks or the prominent groups of peaks are all centred at about 22 Hz. This latter point and the points discussed above all suggest that, at the present

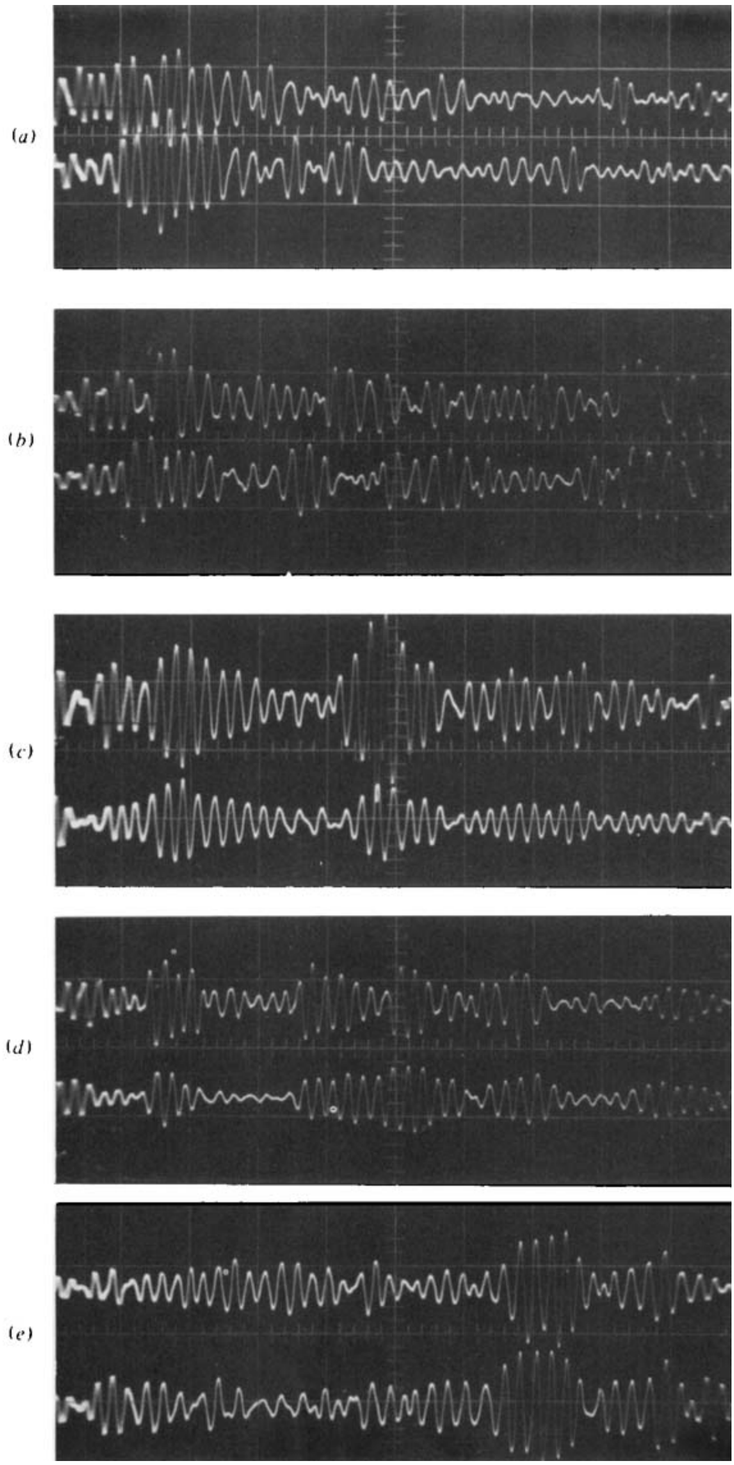


FIGURE 10. Microphone signals recorded simultaneously two at a time. Band-pass filter, 20–25 Hz. Time axis, 0·2 s/cm. (*a*) and (*b*), microphones at 90° to the wind; (*c*), (*d*) and (*e*), at 135° and 45°. (*a*) Upper, microphone 8; lower, 9. (*b*) Upper, 6; lower, 7. (*c*) Upper, 8; lower, 9. (*d*) Upper, 4; lower, 3. (*e*) Upper, 10; lower, 8. $R \simeq 1{\cdot}8 \times 10^5$.

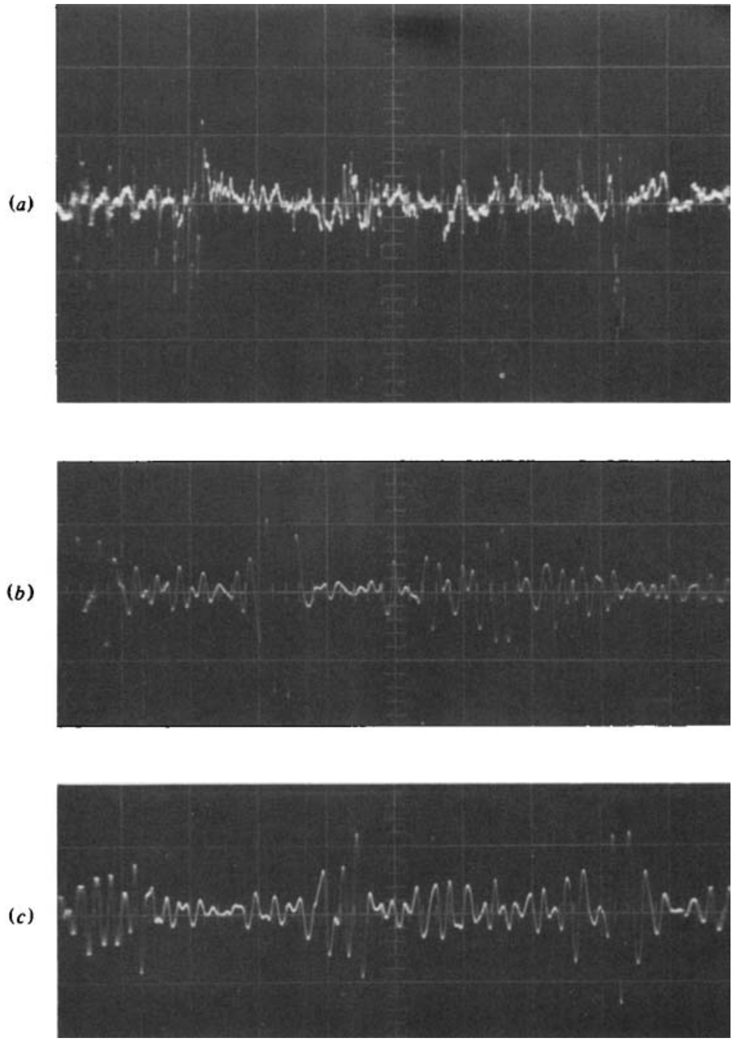


FIGURE 11. Hot-wire signals, five diameters downstream of the cylinder on the wake centre plane, $1d$ from the tip. Time axis, 0·1 s/cm. (*a*) Low-pass filter, 10^3 Hz. (*b*) Band-pass filter, 30–50 Hz. (*c*) Same as in (*b*). $R \simeq 1{\cdot}8 \times 10^5$.

Reynolds number, a vortex-shedding process does indeed take place in the vicinity of the cylinder tip, but that it does only intermittently, that is at some intervals of time but not at others. The evidence also suggests that, when the shedding occurs, it does at one single frequency in the tip region, but that this frequency is not the same at different times of shedding and is, most of the time, lower than the frequency that would be observed if the cylinder were infinite.

In order to check further on the above hypothesis, the velocity signals on the wake centre plane, five diameters downstream of the cylinder, and at different spanwise positions from the tip, were examined visually (see figures 11 and 12, and compare). What was found was that the velocity fluctuations at zero distance from the tip resemble, most of the time, those in the free stream, those at one diameter away are free-stream-like at times and turbulent with high intensity at others, and finally those at

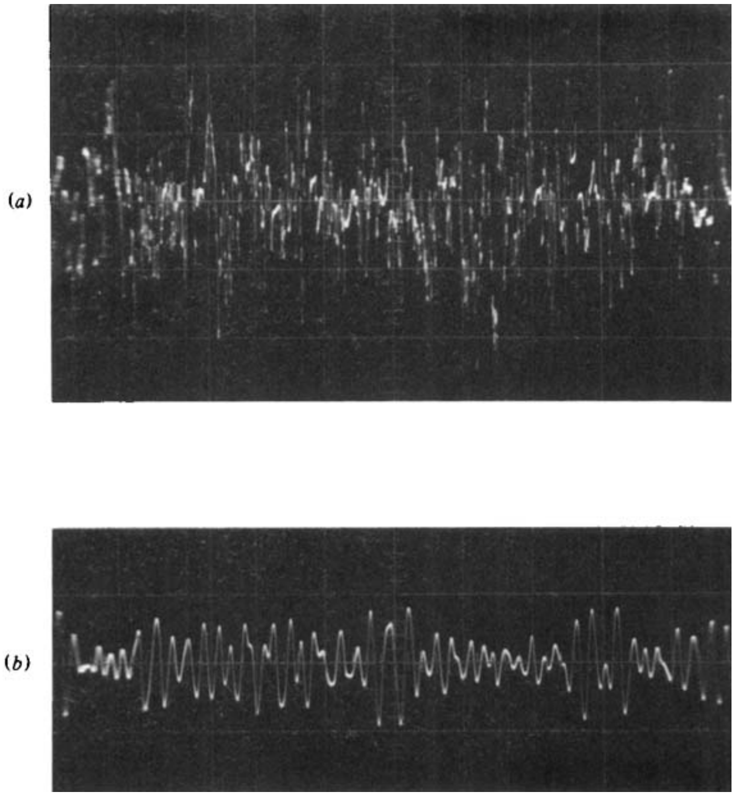


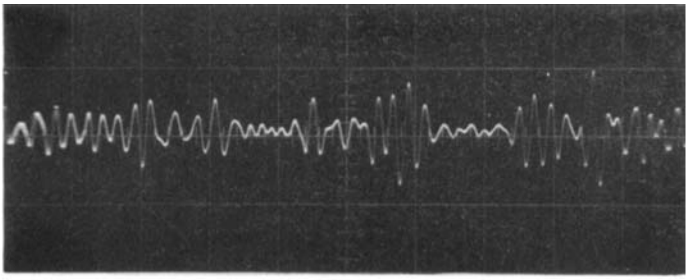
FIGURE 12. Hot-wire signals, five diameters downstream of the cylinder on the wake centre plane, $2d$ from the tip. Time axis, 0·1 s/cm. (a) Low-pass filter, 10^3 Hz. (b) Band-pass filter, 30–50 Hz. $R \simeq 1{\cdot}8 \times 10^5$.

two diameters from the tip are consistently turbulent with high intensity. This implies that the boundary of the wake, in the spanwise direction, is oscillating. Such oscillation could be a manifestation of fluctuations in the magnitude of the downwash, which could mean, since the downwash is due to a flow over the top of the cylinder, that the portion of the cylinder near the tip sees a free-stream velocity that is changing with time, thus leading to the instability of the shedding process and the presence of more than one frequency of shedding. In other words, it could be that the flow in the tip region is constantly attempting to lock itself into a shedding regime – only to find, when it does manage to do so, that the regime reached is unstable – and consequently goes into a chaotic state for a while before repeating the process.

Before proceeding further we should point out that this instability of the shedding regime and the intermittency in the shedding that results from it could be due to the disturbances created by the presence of the microphones on the surface of the cylinder. In our discussion above we have linked the intermittency to the fluctuations in the downwash, but it is possible that both features are caused by disturbances on the surface of the cylinder in the tip region where the microphones are located; the present Reynolds number is a high subcritical one and is very close to the critical value where surface roughness is known to disturb the shedding process.† In such a case, the re-

† Note, however, that this does not apply in the region away from the tip where, as we will see further below, the shedding regime is a conventional subcritical one.

(a)



(b)

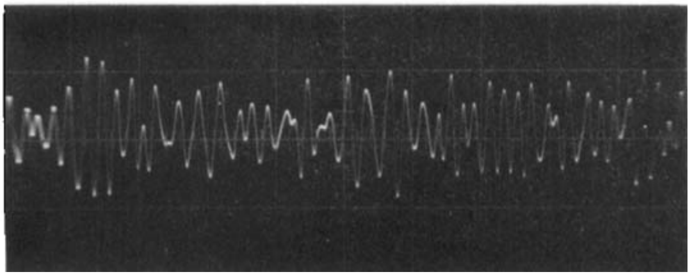


FIGURE 13. Microphone signals at 180° from the front stagnation point. Band-pass filter, 30–50 Hz. Time axis, 0·1 s/cm. (*a*) Signal from microphone 10; (*b*) signal from 6. $Re \simeq 1{\cdot}8 \times 10^5$.

current breakdown in the shedding process, due to the surface disturbances, would cause the downwash to fluctuate.

One additional comment concerning the effect of the surface microphones on the flow is in order. As was indicated earlier, the pressure measurements at different angular positions to the wind were made by rotating the cylinder around its axis in steps of 45°. On the other hand, it is well-known that in the subcritical range of Reynolds number the boundary layer's separation point oscillates between 80° and 90° from the front stagnation point. Thus it is very likely that in the 90° position the microphones had a particularly disturbing effect on the flow, whence follows the peculiar character of the spectra in figure 8. However, we feel that this effect could have been only local since the wake-velocity spectra (to be discussed shortly), which are the results of measurements made with the microphones at 90° to the wind, greatly resemble the pressure spectra at 45° and 135°.

Other features of the flow that are evident in figures 7–9 include the manner in which the energy associated with vortex shedding, in the surface-pressure fluctuations, changes from one location to another. As figure 7 shows, that energy has a maximum level, at 135° to the wind, at about one diameter from the tip, while farther down along the span it reaches a level that is the same at both 45° and 135° to the wind (see figure 9). This is in contrast to the situation at $R = 0{\cdot}85 \times 10^5$, where the energy associated with vortex shedding increases from a minimum near the tip and maintains, away from the tip and at 45° and 135°, a relationship that is roughly the same as the one that prevails in the case of an infinite cylinder. Note also the marked difference in the level of low-frequency fluctuations near the tip at 90° and 135° to the wind. As for

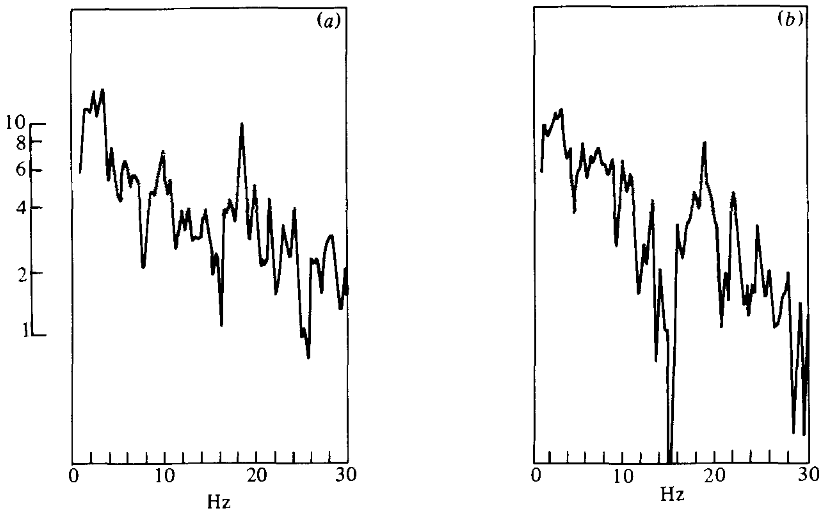


FIGURE 14. Frequency spectra of velocity fluctuations, $1d$ downstream of the cylinder, $\frac{1}{2}d$ from the tip, and $1d$ off the wake centre plane: (*a*) at the position of the reference hot wire; (*b*) at the mirror image of the reference-hot-wire position with respect to the wake centre plane (see figure 2). The unit on the vertical axis is arbitrary but the scale is the same for all spectra in figures 10 and 11. $R \simeq 1{\cdot}8 \times 10^5$.

the spectra of the surface pressure at 0° and 180° to the wind, they contain no particularly significant information, except the fact that the fluctuations are random and of low intensity at 0° and random and of high intensity at 180°, and therefore will not be shown here. (See, however, figure 13, which shows that when a band-pass filter is applied to the raw signals the double-shedding frequency can be seen clearly.) We will not show, either, the spectra of the pressure fluctuations on the flat end of the cylinder, since they too show that these fluctuations are only of random character.

Figures 14 and 15 show respectively the frequency spectra of the velocity fluctuations near the tip of the cylinder and at several locations five diameters downstream. The spectra in figure 15 correspond to measurements on one side of the wake only; the data taken on the other side were used only for cross-correlation purposes. As for the spectra of the velocity fluctuations on the wake centre plane, they reveal no particularly important information, except the fact that the fluctuations there are random, and therefore will not be shown. (Note, on the other hand, the double-shedding-frequency component in figures 11 and 12.)

As is clear from the spectra of figures 14 and 15, the velocity fluctuations in the wake, like those of the surface pressure, contain more than one prominent frequency component. Groups of prominent peaks or prominent peaks accompanied by side peaks, rather than single isolated peaks, appear in most of these spectra. However, one additional feature is clearly evident in the spectra of figure 15, which correspond to the velocity fluctuations at various spanwise locations, and has to do with the shift in the frequency corresponding to the highest peak toward higher values, as the distance from the tip increases. This frequency is equal to 18 Hz at one diameter from the tip, 18·8 Hz at two diameters, 19·6 Hz at three diameters, 21·6 Hz at five diameters, and 23·3 Hz at seven diameters from the tip. It is clear that in the last position the frequency corresponding to the highest peak has reached the value of the vortex-shedding frequency

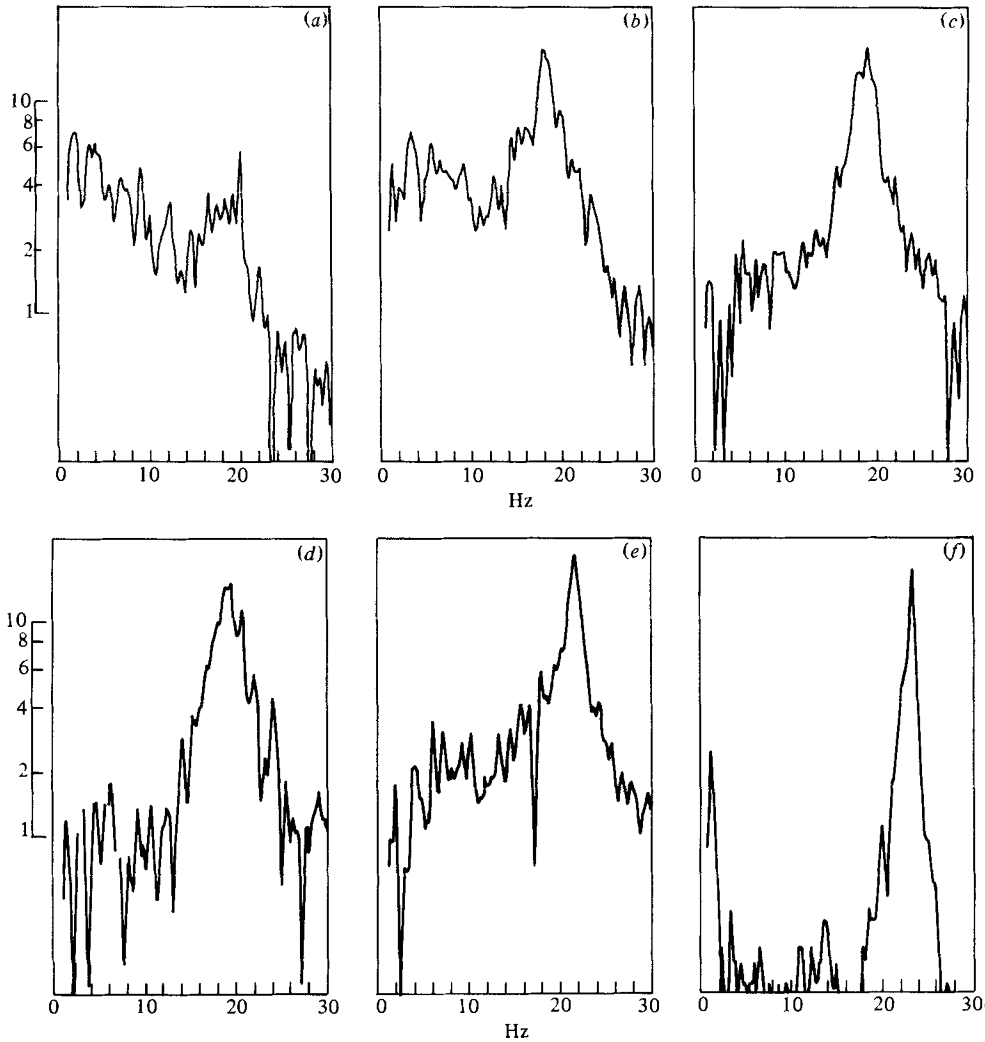


FIGURE 15. Frequency spectra of velocity fluctuations, $5d$ downstream of the cylinder, $2d$ off the wake centre plane (*a*) $0d$, (*b*) $1d$, (*c*) $2d$ (*d*) $3d$, (*e*) $5d$, (*f*) $7d$ from the tip. $R \simeq 1{\cdot}8 \times 10^5$.

that would be observed if the cylinder were infinite. Visual inspection of the hot-wire signals corresponding to the spectra of figure 15 reveals that the velocity fluctuations at zero distance from the tip are of very low level and free-stream-like most of the time. The visual inspection also reveals that at the other locations these fluctuations are intermittently periodic and improve in regularity with increasing distance from the tip.

That the spectral peaks in figures 14 and 15 correspond to a vortex-street-like motion in the wake will be discussed further below. Here, it is clear from the above, and as was argued in §3.1, that the flow downstream of the cylinder is divided, in the spanwise direction, into separate vortex cells, with the cells toward the tip having fewer vortices than those farther away. It is also clear, in view of the fact that the vortex shedding from the cylinder is everywhere, in the tip region, centred at 22 Hz, that the cells form downstream of the formation region. It should also be noted that the appearance of more than one prominent peak, or the appearance of prominent peaks with side peaks,

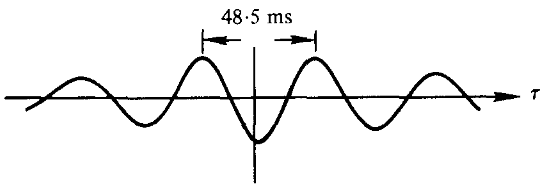


FIGURE 16. Cross-correlation function of filtered signals from reference hot-wire probe and the traversing hot-wire probe in the position symmetric to that of the reference probe with respect to the wake centre plane. Signal from traversing probe delayed. $R \simeq 1{\cdot}8 \times 10^5$.

in the velocity spectra is due not only to the changes that occur in the shedding frequency in the vicinity of the tip but also to the presence of the cells: a hot-wire probe at a given spanwise location senses not only the frequency of passage of vortices through that location but also the frequency of passage of vortices through a nearby location.

As for the spectra shown in figure 14, which correspond to the velocity fluctuations in the immediate vicinity of the tip, they indicate that the vortex-shedding component is not dominant there; this, despite the fact evident in figure 9 that the shedding extends up to $0{\cdot}6d$ from the tip. The velocity fluctuations in the immediate vicinity of the tip seem to reflect the motion of the vortex cells at nearby points. Examination of the raw signals reveals that a periodic component is there at times but not at others, and when it is there it is 180° out of phase between the two signals. This is also reflected in the cross-correlation function shown in figure 16 and obtained after band-pass filtering the signals between 17 and 25 Hz. An 'average' frequency, i.e. a frequency that is higher than the lowest common frequency and lower than the highest common frequency in the two signals, can be computed from the cross-correlation function in figure 16, and is equal to 20·6 Hz.† Clearly, this value is lower than the 22 Hz at which the vortex shedding in the tip region is centred.

Figure 17 shows the cross-correlation functions of the signal from the reference hot wire with the signal from the traversing hot wire in three different positions five diameters downstream of the cylinder on each side of the wake. These cross-correlations were made after having band-pass filtered the signals between 17 and 25 Hz. Since both the reference-hot-wire signal and the signal from the traversing hot wire in any of the positions in question contain more than one frequency component, and since each of the frequency components in common between the two signals, whether it is due to the motion of the vortices in a nearby cell or to the shedding regime in progress in a given interval of time, takes a particular time to travel from the position of the reference probe to the position of the traversing probe, the phase difference associated with different frequency components must necessarily differ from one component to another. Therefore it appears that it would be very difficult to extract phase information from the cross-correlation functions of figure 17. However, if the flow in the wake is that of a vortex-street-like system, what goes on on one side of the wake centre plane must also go on the other side, with a phase lag of 180°, and this should be reflected in the cross-correlation functions. Indeed, comparison of figures 17 (*a–c*) with figures 17 (*a′–c′*)

† When two signals containing more than one frequency component are cross-correlated, only the common frequency components give a net contribution. Furthermore, if the phase difference is the same for all the common frequency components, this phase difference will be reflected in the central period between the two maxima bracketing $\tau = 0$ of the resulting cross-correlation, and an 'average' frequency can be computed on the basis of the central period.

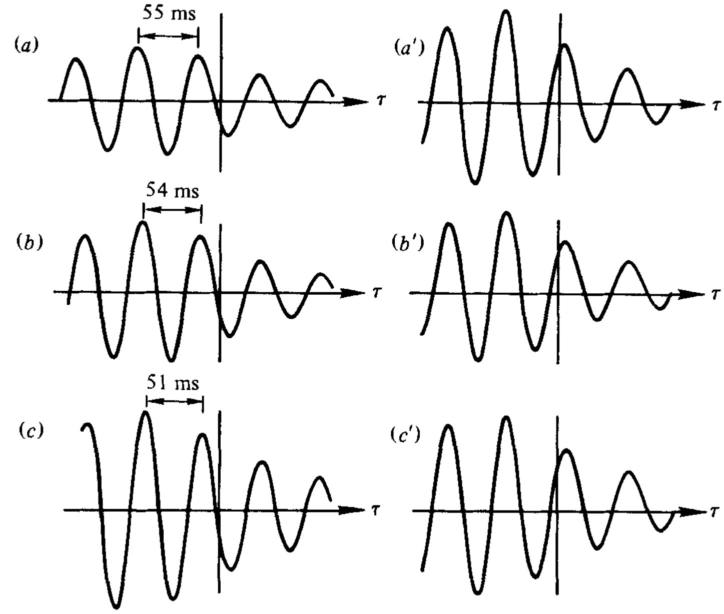


FIGURE 17. Cross-correlation functions of filtered signals from reference hot-wire probe and the traversing hot-wire probe in the downstream position $5d$ from the cylinder, $2d$ off the wake centre plane (*a*) $1d$, (*b*) $2d$, (*c*) $3d$ from the tip on one side of the wake and (*a′*) $1d$, (*b′*) $2d$, (*c′*) $3d$ from the tip on the other side of the wake. Signal from traversing probe delayed. $R \simeq 1{\cdot}8 \times 10^5$.

reveals that a phase difference of roughly 180° does exist, on the average, between the velocity fluctuations at two symmetric points with respect to the wake centre plane. It may also be noted that the frequency computed using the period indicated in figures 17 (*a*–*c*) is very close to the frequency of the highest peak in the corresponding spectra in figures 15 (*b*–*d*); it is equal respectively to 18·2 Hz, 18·5 Hz, and 19·6 Hz.

To conclude the present subsection, we suggest, on the basis of the results discussed above, that the flow past the cylinder at $R \simeq 1{\cdot}8 \times 10^5$ is composed of two main regions: a tip region that extends up to a few diameters from the tip, and another region that covers the remaining part of the flow. In the latter region the flow is as it would be if the cylinder were infinite, while in the tip region a lower-Reynolds-number vortex-shedding regime takes place that is, nonetheless, unstable and has in addition a cellular structure in the wake.

3.3. *The case* $R \simeq 7{\cdot}7 \times 10^5$ ($q = 80\,\mathrm{lbf/ft^2}$)

As in the previous case, the data recorded at $q = 80\,\mathrm{lbf/ft^2}$ correspond, rather, to a range of values of R. This range extends from about $7{\cdot}45 \times 10^5$ to about $7{\cdot}96 \times 10^5$ and is well into the supercritical range by infinite-cylinder standards. Normally, no vortex shedding is observed in the wake of an infinite cylinder in the supercritical range, and when a peak appears in the spectra of, say, the velocity fluctuations it is broad-band at best. The values of S that one finds in the literature are based on the central frequency of the peak and, in fact, show considerable scatter. Bearman (1969) suggests that a value for S between 0·4 and 0·45 is to be expected in the range of R between 6×10^5 and $1{\cdot}6 \times 10^6$. However, if the data of Relf and Simmons (see Bearman 1969), as well as the trend apparent in the data of Delany & Sorenson (1953) and Cometta (1957) are taken into account, a value between 0·3 and 0·42 would seem more appropriate for

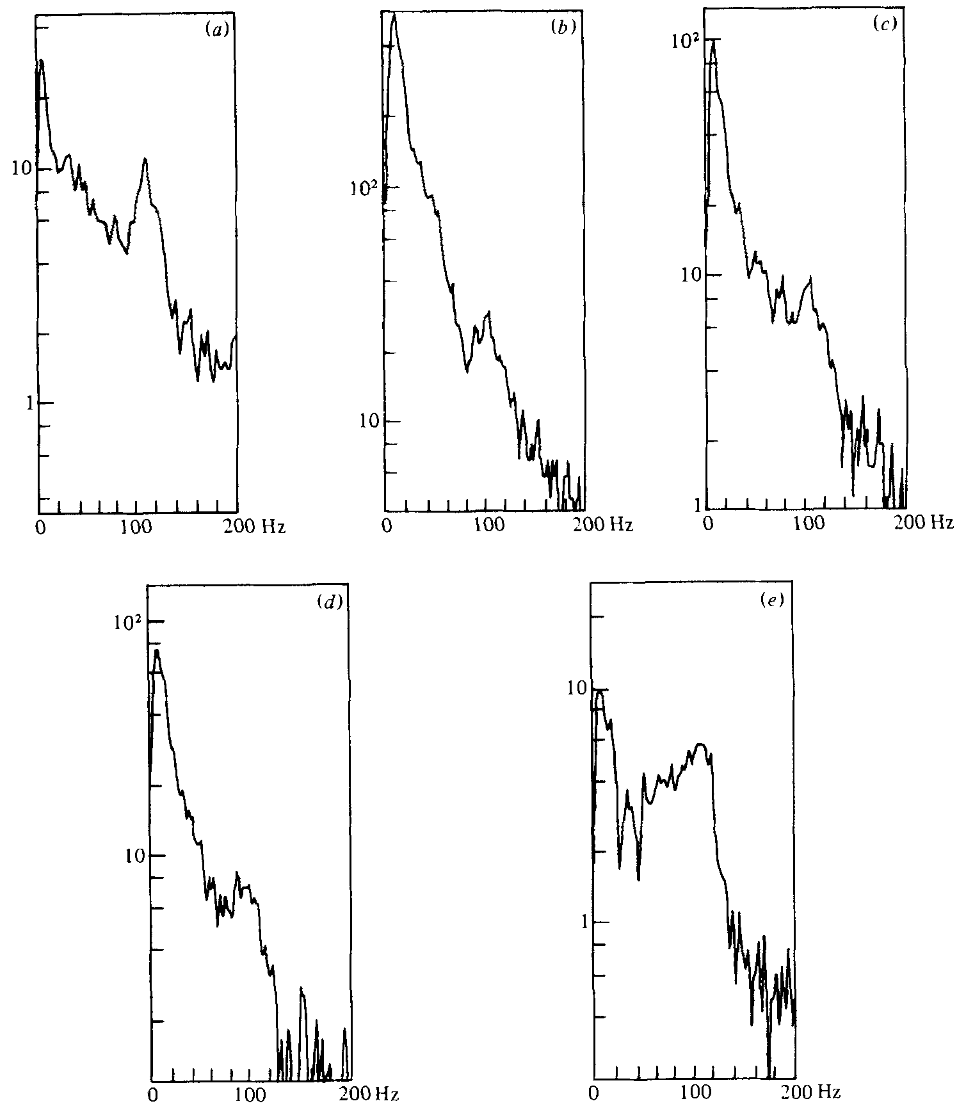


FIGURE 18. Frequency spectra of surface-pressure fluctuations at 90° to the wind and (a) 0·29d, (b) 1·04d, (c) 1·79d, (d) 2·54d, (e) 3·29d from the tip. The unit on the vertical axis is arbitrary but is the same for all spectra. $R \simeq 7{\cdot}7 \times 10^5$.

the conditions of the present experiment. This means that in the present case a central frequency between about 160 Hz and 220 Hz would be observed if the cylinder were infinite.

Figures 18, 19, and 20 show respectively the frequency spectra of the surface-pressure fluctuations at 90° to the wind, the frequency spectra of the velocity fluctuations near the tip of the cylinder, and the frequency spectra of the velocity fluctuations five diameters downstream. The spectra shown are portions of original spectra that were computed in the range between 0 and 1000 Hz, with a frequency resolution of 5 Hz.

A peak centred at a frequency between 105 and 110 Hz, corresponding to a value of S of about 0·2, is clearly evident in both the surface-pressure and the wake-velocity

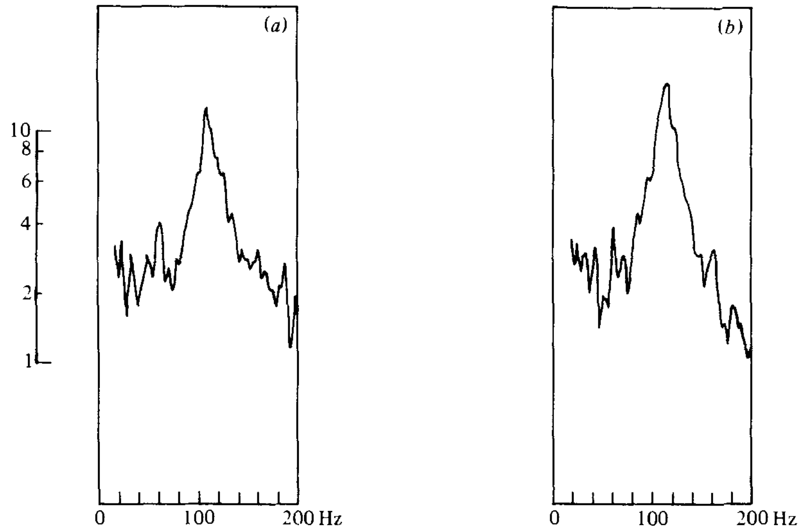


FIGURE 19. Frequency spectra of velocity fluctuations, $1d$ downstream of the cylinder, $\frac{1}{2}d$ from the tip, and $1d$ off the wake centre plane: (*a*) at the position of the reference hot wire; (*b*) at the mirror image of the reference-hot-wire position with respect to the wake centre plane (see figure 2). The unit on the vertical axis is arbitrary but the scale is the same for all spectra in figures 19 and 20. $R \simeq 7{\cdot}7 \times 10^5$. *Note:* the peak at 60 Hz is due to the effect of the power line on the hot-wire signal.

spectra, particularly in those that correspond to locations toward the tip. With increasing distance from the tip this peak gradually disappears in the spectral background and the spectra become typical of those that are observed with infinite cylinders at supercritical values of the Reynolds number. Furthermore, as is clear in figure 18, the energy in the surface-pressure fluctuations that are associated with this spectral peak is maximum at about one diameter from the tip. Note, also, the high level of pressure fluctuations in the low-frequency range, which again is maximum at about one diameter from the tip.

As was confirmed by filtering and cross-correlating the pressure signals from either side of the cylinder with respect to the direction of the free stream (the signals were band-pass filtered between 80 and 120 Hz), the spectral peak in figure 18 corresponds to a vortex-shedding process; a phase lag of 180° was clearly evident in the cross-correlation functions involving signals from the upper and lower sides of the cylinder, despite the broad-band character of the shedding. Visual inspection of the raw signals did not prove helpful in the present case; however, no intermittency of the type that was found in the signals at $R \simeq 1{\cdot}8 \times 10^5$ was evident at the present value of R.

The pressure spectra (not shown here) that correspond to the angular positions 45° and 135° to the wind do not contain any useful information, except the fact that the level of fluctuations at 135° is lower than that at 90°; this is unlike the situation at $R \simeq 1{\cdot}8 \times 10^5$. As for the pressure fluctuations at 0° and 180° to the wind, and on the flat end of the cylinder, the corresponding spectra show that they are highly irregular.

Figure 21 shows the cross-correlation function of the reference-hot-wire signal with the signal from the traversing hot wire in the near-tip position (both signals were band-pass filtered between 80 and 120 Hz). As is clear from this and the cross-correlation functions (not shown) involving the reference hot wire and the traversing hot wire in

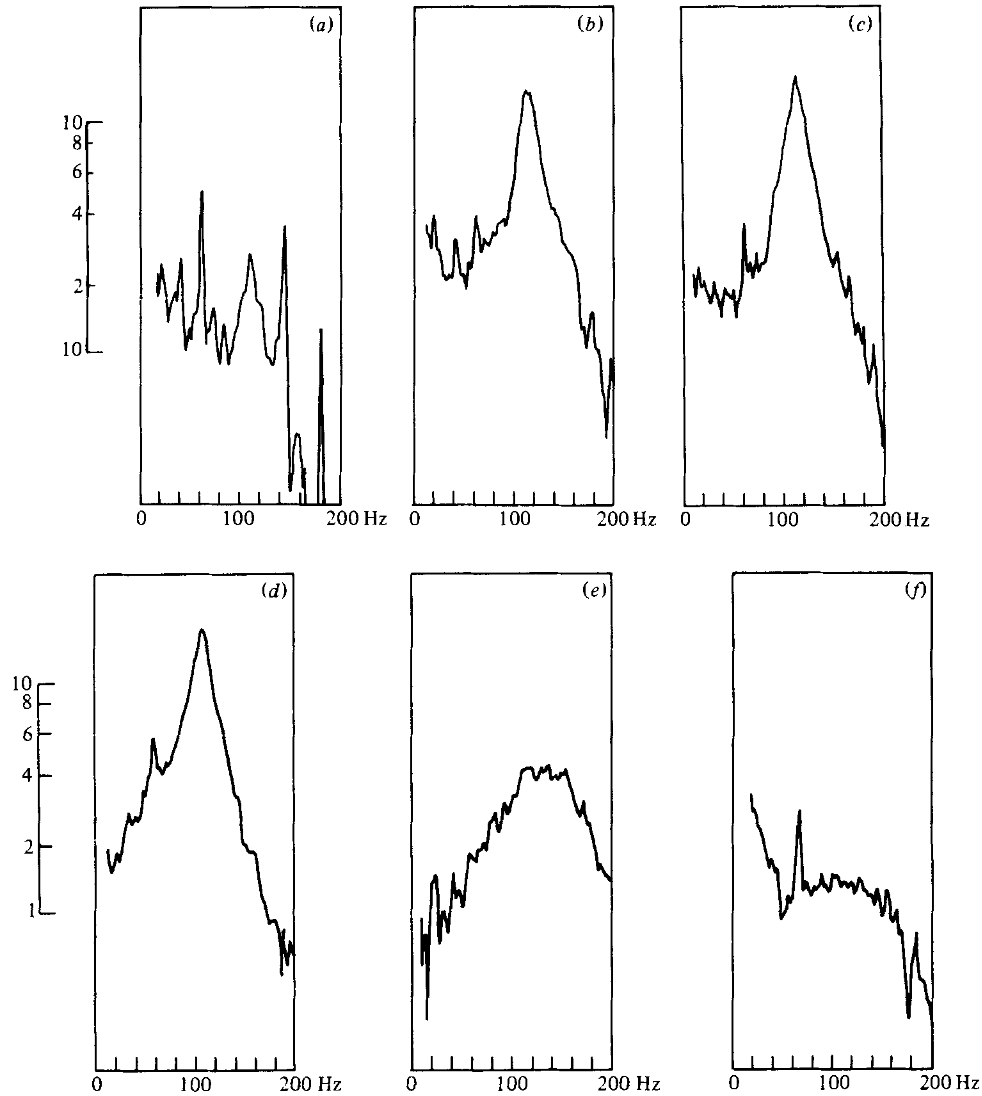


FIGURE 20. Frequency spectra of velocity fluctuations, $5d$ downstream of the cylinder, $2d$ off the wake centre plane: (*a*) $0d$, (*b*) $1d$, (*c*) $2d$, (*d*) $3d$, (*e*) $5d$, (*f*) $7d$ from the tip. $R \simeq 7 \cdot 7 \times 10^5$. *Note:* the peak at 60 Hz is due to the effect of the power line on the hot-wire signal.

the downstream positions (except at 5 and 7 diameters from the tip), the spectral peaks in figures 19 and 20 correspond to a vortex-shedding process.

The most notable feature of the shedding process, in the tip region, at the present value of R is that, unlike at the previous value, the spectral peaks remain centred at the same frequency throughout the near-wake region. This means that no cellular structure of the type encountered at $R \simeq 1 \cdot 8 \times 10^5$ exists in the present case. Furthermore, the absence of any intermittency in the pressure or velocity signals and the degree of coherence evident from the various cross-correlation functions, such as, for instance, the one shown in figure 21, point to a surprisingly regular, though broad-band, vortex-shedding process. In fact, there is evidence from cross-correlation

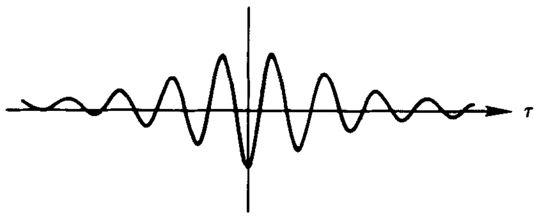


FIGURE 21. Cross-correlation function of filtered signals from reference hot-wire probe and the traversing hot-wire probe in the position symmetric to that of the reference probe with respect to the wake centre plane. Signal from traversing probe delayed. $R \simeq 7{\cdot}7 \times 10^5$.

functions of the type shown in figure 17 that, on the average, the vortices are shed parallel to the cylinder.

To sum up, it appears that the flow past the cylinder at $R \simeq 7{\cdot}7 \times 10^5$ consists of two regions: a tip region where a broad-band, but regular, vortex-shedding process, centred at a Strouhal number characteristic of subcritical values of R, takes place, and a remaining region where a supercritical regime is in effect.

4. Conclusion

The fundamentals of the flow past a finite cylinder with uniform upstream conditions have received comparatively little attention in the past. The three-dimensional flow structure in the tip region and the nature of the unsteady forces that act on the cylinder in the vicinity of the tip pose a challenge to both the fundamental research worker and the practising engineer.

In the present work an attempt was made to uncover some basic patterns in the seemingly complicated flow in the tip region of a finite circular cylinder at three different values of the Reynolds number, two subcritical and one supercritical. The main conclusions that can be made are as follows.

(1) Vortex shedding from a finite cylinder in a cross flow extends up to a short distance from the tip.

(2) In the tip region the vortices are shed coherently at one single frequency. The observed gradual decrease toward the tip in the frequency of passage of vortices at some distance downstream is due to a separate phenomenon that occurs downstream of the formation region and results in the formation of cells in the wake's vortex street.

(3) Generally, the shedding regime in the tip region corresponds to a value of the Reynolds number that is lower than the nominal one. In particular, a subcritical-regime vortex shedding in the tip region may well coexist with a supercritical flow on the main portion of the cylinder.

(4) Under certain circumstances, which deserve further study, the shedding regime in the tip region may be unstable, in which case the flow is intermittent and the shedding frequency assumes different values at different time intervals.

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